ELEMENTARY ARITHMETIC

OF THE

OCTIMAL NOTATION

BY

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PREFACE.

History shows that many unsuccessful attempts have been made to discover a system of notation which would be simple and effective, and we wonder that a satisfactory basis has not been found. It is the more surprising, when we consider that every part of the universe suggests multiplication or division by two as a basis from which to start the building of a system of notation. This principle is exposed in the attempt to provide a Decimal currency, for the result is an Octimal coinage, viz., the Dollar, half, and quarter. And so, also, the attempt to decimalize the circle, for the quadrant must be found by the same process. In fact, there is nothing in the universe to suggest the use of ten as a radix. So many obstacles stand in the way of the successful application of the Decimal Notation, that it is astonishing it has not not been abandoned long ago.

The Octimal System will be the means of considerably reducing the work in schools, by rendering much of the work unnecessary, which, owing to the defects of the Decimal System, is now required, and, in addition, will make arithmetical work a pleasure instead of a drudgery.

Its introduction need cause no confusion, for the present units in use will be retained; viz., the Inch, the Pound, and the Dollar, while the methods of manipulation are the same as those already practiced.

Any attempt to perpetuate the use of the Decimal Sys-

tem is nothing short of a crime against humanity, since it fails in every department.

The principle of "metric" arrangement has been adopted in this work, and followed out to its logical conclusion, and the desired result has been attained,—a perfect system of notation, and arrangement of weights and measures.

The Arithmetic is therefore offered in the confidence that it will prove an educational factor of great value, and forever place the science of numbers on a sound basis.

THE AUTHOR.

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ELEMENTARY ARITHMETIC

OF THE

OCTIMAL NOTATION.

SECTION 1.

ARITHMETIC.

ARITHMETIC teaches us the use of Numbers.

A Unit, or One, is any single object or thing; as, an orange, a tree.

A Whole Number, or an Integer, is a Unit, or One, or a collection of ones. If a boy, for instance, have one orange, and then another is given to him, he will have two oranges; if another be given to him, he will have three oranges; if another, four oranges, and so on. One, two, three, four, etc., are called Whole Numbers, or Integers.

Notation is the art of writing any number in figures.

The Octimal Notation is the method of expressing numbers by means of the following radix of figures:—

	1	2	3	4	5	6	7	0	
called	one,	two,	three,	four,	five,	six,	seven,	eights,	or cipher.

representing (if we express a unit by a dot; thus, .),

.	..	...	....	.....	......	....	....
or one unit	or two units	or three units	or four units	or five units	or six units	or seven units	or eights units

The first seven figures have a fixed value. The value of the last depends upon whether it is prefixed by another figure or not, and also the value of the prefixed figure. When standing alone, without an index, it has no value; but when prefixed by the Index, one, its value is eight; if

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by two, two eights or two-et, etc., etc. Any of the figures from one to seven, when prefixed to the cipher (thus, 10), indicates that the radix has been named over as many times as the prefixed figure has value in units. Thus if the radix has been counted through once, the figure 1 is to be prefixed; and if twice, the figure 2; and if three times, the figure 3, and so on until all the figures have been prefixed.

The order of progression in the Octimal System of Notation is in powers of eight; that is to say, that when eight has been counted once, that fact is to be recorded by prefixing the figure 1 to the cipher (thus, 10), so that while the figures of fixed value stand alone, they only stand for the number of units they represent; but the addition of the cipher denotes that the number to which it is affixed has been counted as many eight times. Thus 70 denotes that 7 has been counted eight times, or 8 seven times, and the figure prefixed to the cipher to record that fact.

When the radix has been counted through eight times, the figures which stand for one eight are to be prefixed to a new cipher; thus, 100, one eight eight, the name of this combination being abbreviated to etred. All numbers greater than eight, but less than etred, will be represented by two figures; thus 11 signifies that eight has been counted once, because the figure 1 appears as the second figure to the left, the place to be occupied by the figures which represent the number of eights which have been counted, and the figure 1 to the right denotes that one more than eight has been counted; and as the figure 1 must occupy the units' place in the number, the cipher must be replaced by that figure.

We therefore understand that the first place to the left of a point belongs to the ones of units counted up to seven and the cipher when the number is a multiple of eight, and the second place to the left belongs to the eights of units up to the seventh eight, and a cipher when the number is a multiple of eight, and so on.

EXAMPLE.

Thus one	is written		1.
eight	"	"	10.
etred	"	"	100.
etand	"	"	1,000.
eight etand	"	"	10,000.
etred etand	"	"	100,000.
million	"	"	1,000,000.
eight million	"	"	10,000,000.
etred million	"	"	100,000,000.
billion	"	"	1,000,000,000.

The reversal of positions of figures alters their value.

Thus one eighth	is written		.1,	or point		1.
one etredth	"	"	.01,	"	"	01.
one etandth	"	"	.001,	"	"	001.
one eight etandth	"	"	.0001,	and so on.		
one etred etandth	"	"	.00001.			
one millionth	"	"	.000001.			
one eight millionth	"	"	.0000001.			
one etred millionth	"	"	.00000001.			
one billionth	"	"	.000000001.			

From the above table we see that dividing any number

into periods of three figures each, beginning at the right hand, the names of those periods will be—

First	period,	Units.
Second	"	Etands.
Third	"	Millions.
Fourth	"	Billions.
Fifth	"	Trillions.

The names of the places in each of these periods are the same.

Namely,	First	place,	Units.
	Second	"	Eights.
	Third	"	Etreds.

The following plan is recommended to enable the pupil to write in figures any number dictated by the teacher:—

Let the pupil write on his slate a number of ciphers; thus, 000,000,000,000, marking them off into periods of three places each from the right;

Put U	over	the first	period	for	Units;
E	"	" second	"	"	Etands;
M	"	" third	"	"	Millions;
B	"	" fourth	"	"	Billions;

B M E U

and so on; thus, 000,000,000,000. Then when a number is dictated to a pupil, all he has to do is to put each figure in its proper place and fill up vacancies with 0's.

Exercises.

Write the following numbers in figures:—

(1) One, four, six, three, seven, eight.

(2) Et-seven, two-et, four-et one, six-et six, two-et two.

(3) Seven-et three, etred and et-one, three etred and two-et two.

(4) Etand six etred and four-et three, three etand and five.

(5) Seven-et six etand three etred and five-et six.

(6) Two-et three million seven etred and six-et etand and four.

(7) Four etred million three etred etand five etred and two-et.

Thus two etand and three will be written thus in figures: 2,003;

Six-et etand, three etred and two: 60,302;

Four etred and three-et etand, six etred and two-et one: 430,621;

Six etred and one millions, four etred and seven: 601,-000,407.

Numeration is the art of writing in words the meaning of any number which is already given in figures; thus,

35 means three eights and five units, or three-et five;

506 means five etreds no eights and six units, or five etred and six;

0604 means no etands six etreds no eights, or six etred and four;

3,251,630 means three millions, two etred and five-et one etands, six etred and three-et.

Exercises.

Write in words the meaning of—

(1) 2, 4, 6, 1, 7, 12, 16, 40, 23, 75, 67, 32, 44.
(2) 63, 21, 17, 35, 52, 47, 77, 34, 16, 75.
(3) 102, 165, 143, 371, 263, 542, 764, 463.
(4) 1423, 3764, 4321, 5635, 7352, 6473.
(5) 53742, 64371, 42536, 57324, 65437.
(6) 140526, 330000, 4773254, 4613025.

SECTION 2.

ADDITION.

Addition is the method of finding a number which is equal to two or more numbers of the same kind taken together.

The numbers to be added are called Addends.

The Sum, or Amount, is the number so found.

ADDITION TABLE.

2 and

1	make	3, Three.
2	"	4, Four.
3	"	5, Five.
4	"	6, Six.
5	"	7, Seven.
6	"	10, Eight.
7	"	11, Et-one.
10	"	12, Et-two.

3 and

1	make	4, Four.
2	"	5, Five.
3	"	6, Six.
4	"	7, Seven.
5	"	10, Eight.
6	"	11, Et-one.
7	"	12, Et-two.
10	"	13, Et-three.

4 and

1	make	5, Five.
2	"	6, Six.
3	"	7, Seven.
4	"	10, Eight.
5	"	11, Et-one.
6	"	12, Et-two.
7	"	13, Et-three.
10	"	14, Et-four.

5 and

1	make	6, Six.
2	"	7, Seven.
3	"	10, Eight.
4	"	11, Et-one.
5	"	12, Et-two.
6	"	13, Et-three.
7	"	14, Et-four.
10	"	15, Et-five.

6 and			7 and		
1	make	7, Seven.	1	make	10, Eight.
2	"	10, Eight.	2	"	11, Et-one.
3	"	11, Et-one.	3	"	12, Et-two.
4	"	12, Et-two.	4	"	13, Et-three.
5	"	13, Et-three.	5	"	14, Et-four.
6	"	14, Et-four.	6	"	15, Et-five.
7	"	15, Et-five.	7	"	16, Et-six.
10	"	16, Et-six.	10	"	17, Et-seven.

The sign + called plus, placed between two numbers means that the two numbers are to be added together; thus $2+2$ make 4, and $2+2+1$ make 5.

The sign = placed between two sets of numbers means that they are equal to one another; thus, $2+2=4$, and $2+2+1=4+1$, and $2+2+1=5$, and $4+1=5$. The sum of $2+2+1$ and $4+1$ are equal.

The sign $\therefore$ means therefore.

Example 1.—Find the sum of 3, 5, and 2. We add thus: 3 and 5 make 10, 10 and 2 make 12; $\therefore$ the sum of $3+5+2=12$.

Example 2.—Add together 5, 7, 2, 6, and 3. 5 and 7 make 14, 14 and 2 make 16, 16 and 6 make 24, 24 and 3 make 27; $\therefore$ $5+7+2+6+3=27$;

or thus:—

Add—	(1)	(2)	(3)	(4)	(5)
	3	4	3	4	7
	5	6	7	0	3
	2	2	1	1	2
	7	5	0	2	5
	6	3	6	5	1
	27	24	21	14	22

RULE FOR ADDITION.

Rule.—Write down the given numbers under each other, so that units may come under units, eights under eights, etreds under etreds, and so on; then draw a line under the lowest number.

Find the sum of the column of units; if it be less than eight, write it down under the column of units, below the line just drawn; but if it be greater than eight, or a multiple of eight, then write down the surplus of units under the units' column, and carry to the column of eights the remaining figure or figures, which of course will be eights.

Add the column of eights and the figure or figures you carry as you have added the column of units and treat its sum in the same way you have treated the column of units.

Treat each succeeding column in the same way.

Write down the full sum of the last column on the left hand.

The entire sum thus obtained will be the sum or amount of the given numbers.

Example 1.—Add together 46, 53, and 164.

By the Rule.

```
 46
 53
164
---
305
```

Method of Adding.—4 and 3 are 7, 7 and 6 are 15. Write down 5 and carry 1, because the sum of the units' column is one eight units and 5 units over.

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Then 1 and 6 are 7, 7 and 5 are 14, 14 and 4 are 20. Write down 0 under the eights' column and carry 2. Then 2 and 1 are 3. Write down 3 under the etreds' column, which completes the calculation.

EXAMPLES.

Add—	(1)	(2)	(3)	(4)	(5)	(6)
	11	15	7	21	35	56
	13	6	16	32	47	44
	21	23	14	16	23	73
	45	46	41	71	127	215

(7)	(10)	(11)	(12)	(13)	(14)
52	76	134	73	321	530
73	121	62	252	763	627
104	32	4	134	420	46
251	251	222	501	1724	1425

(15)	(16)	(17)	(20)
7350	1276	6325	537654
14263	43502	52403	263421
54301	73025	73526	437562
100134	140025	154456	1463057

SUBTRACTION.

SUBTRACTION is the method of finding what number remains when a smaller number is taken from a greater number.

The number so found is called the remainder, or difference.

The sign —, called Minus, placed between two numbers, means that the second number is to be subtracted from the first number; thus $7-3$ means that 3 is to be subtracted from 7; $\therefore 7-3=4$.

RULE FOR SUBTRACTION.

RULE.—Write down the less number under the greater number, so that units may come under units, eights under eights, etreds under etreds, and so on; then draw a straight line under the lower number.

Take, if you can, the number of units in each figure of the lower number from the number of units in each figure of the upper number which stands directly over it, and place the remainder under the line just drawn, units under units, eights under eights, etreds under etreds, and so on.

But if the units in any figure in the lower number be greater than the number of units in the figure just above it, then add eight to the upper figure, and then subtract the number of units in the lower figure from the number of units in the upper figure thus increased, and write down the remainder as before.

Add one to the next number in the lower number, and then take this figure thus increased from the figure just above it, by one of the methods already explained.

Go on thus with all the figures.

The whole difference, or remainder, so written down will be the difference, or remainder, of the given numbers.

EXAMPLE 1.—Subtract 3426 from 5345.

By the Rule.

$$\begin{array}{r} 5345 \\ 3426 \\ \hline \text{Difference}=1717 \end{array}$$

METHOD.—6 from 5 I cannot take; $\therefore$ I borrow 10. Now $10+5=15$. I take 6 from 15, which leaves 7. Write down the 7 and carry 1. Next,

1+2=3. Take 3 from 4, which leaves 1. Write down 1 in the eights' place. Now 4 from 3 I cannot take. I borrow 10. Now 10+3=13. 4 from 13 leaves 5. Write down 5 in the etreds' place and carry 1. Next, 1+3=4. Take 4 from 5, which leaves 1. Write down 1 in the etands' place.

NOTE.—The truth of all sums in subtraction may be proved by adding the less number and the remainder together. If the sum has been worked correctly, the two numbers will be equal.

Thus proof of Example 1: Less number + remainder = 3426 + 1717 = 5345, the greater number.

EXAMPLES.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
From	20	24	17	73	21	75	76
Subtract	16	13	3	14	16	37	63
	02	11	14	57	03	36	13

(10)	(11)	(12)	(13)	(14)	(15)	(16)
265	327	630	746	642	525	740
124	146	245	362	137	431	713
141	161	363	364	503	074	025

(17)	(20)	(21)	(22)	(23)	(24)
2653	5341	3421	4763	371246	576356
1241	1432	2675	3652	246517	537642
1412	3707	0524	1111	122527	036514

MULTIPLICATION.

Multiplication is a short method of repeated addition; thus when 2 is multiplied by 3, the number obtained is the sum of 2 repeated 3 times, which sum $= 2+2+2=6$.

The number which is to be repeated or added to itself is called the Multiplicand; thus in the above example, 2 is the multiplicand.

The number which shows how often the multiplicand is to be repeated is called the Multiplier; thus in the above example, 3 is the multiplier.

The number found by multiplication—for instance, 6 in the above example—is called the Product.

The multiplier and multiplicand are sometimes called Factors, because they are factors, or makers, of the product.

The sign $\times$, called Into, or Multiplied by, placed between two numbers, means that the two numbers are to multiplied together.

The following Multiplication Table should be committed to memory:—

Twice			3 times		
1	are	2, Two.	1	are	3, Three.
2	"	4, Four.	2	"	6, Six.
3	"	6, Six.	3	"	11, Et-one.
4	"	10, Eight.	4	"	14, Et-four.
5	"	12, Et-two.	5	"	17, Et-seven.
6	"	14, Et-four.	6	"	22, Two-et two.
7	"	16, Et-six.	7	"	25, Two-et five.
10	"	20, Two-et.	10	"	30, Three-et.

4 times		5 times	
1 are	4, Four.	1 are	5, Five.
2 "	10, Eight.	2 "	12, Et-two.
3 "	14, Et-four.	3 "	17, Et-seven.
4 "	20, Two-et.	4 "	24, Two-et four.
5 "	24, Two-et four.	5 "	31, Three-et one.
6 "	30, Three-et.	6 "	36, Three-et six.
7 "	34, Three-et four.	7 "	43, Four-et three.
10 "	40, Four-et.	10 "	50, Five-et.

6 times		7 times	
1 are	6, Six.	1 are	7, Seven.
2 "	14, Et-four.	2 "	16, Et-six.
3 "	22, Two-et two.	3 "	25, Two-et five.
4 "	30, Three-et.	4 "	34, Three-et four.
5 "	36, Three-et six.	5 "	43, Four-et three.
6 "	44, Four-et four.	6 "	52, Five-et two.
7 "	52, Five-et two.	7 "	61, Six-et one.
10 "	60, Six-et.	10 "	70, Seven-et.

10 times	10 times
10 are 100, Etred.	100 are 1,000, Etand.

1,000 times
1,000 are 1,000,000, Million.

RULE FOR MULTIPLICATION WHEN THE MULTIPLIER IS NOT GREATER THAN SEVEN.

RULE.—Place the multiplier in the units' place under the multiplicand. Draw a line under the multiplier. Multiply each figure of the multiplicand, beginning with the units, by the fig-

ure of the multiplier (by means of the Multiplication Table). Write down and carry as in Addition.

EXAMPLE 1.—Multiply 325 by 2.

By the Rule.

325
2
652

Twice 5 units makes 12 units. Write 2 in the units' place in the product, and carry 1. Next, twice 2 makes 4. Add to the 4 the 1 which was carried, which makes 5. Write down 5 in the eights' place. Next, twice 3 makes 6. Write the 6 down in the etreds' place.

EXAMPLES.

	(1)	(2)	(3)	(4)
Multiply	231	357	426	734
By	2	3	5	6
	462	1315	2556	5450

(5)	(6)	(7)	(10)
7342	6321	52264	563421
5	7	3	4
45152	54667	177034	2716104

RULE FOR MULTIPLICATION WHEN MULTIPLIER IS A NUMBER LARGER THAN SEVEN.

RULE.—Place the multiplier under the multiplicand, units under units, eights under eights, etreds under etreds, and so on; then draw a line under the multiplier. Multiply each figure in the multiplicand, beginning with the units, by the figure in the units' place of the multiplier (by means of the Multiplication Table); write down and carry as in Addition.

Then multiply each figure of the multiplicand, beginning with the units, by the figure in the eights' place of the multiplier,

placing the first figure so obtained under the eights of the line above, the next figure under the etreds, and so on. Proceed in the same way with each succeeding figure of the multiplier.

Then add up all the results thus obtained, by the rule of Addition.

EXAMPLE 2.—Multiply 4362 by 342.

```
   4362
    342
  -----
  10744
 21710
15326
-------
1762644
```

Since 342=300+40+2, we really multiply first by 2, next by 40, and last by 300, and if the sums are placed under each other and added together, the product will appear as shown.

To multiply by eight, all that is necessary is to affix a cipher to the right of the number, and by etred, two ciphers, and so on.

When any other Multiplier terminates with one or more ciphers, multiply by the remaining figures, and to the product add the same number of ciphers; thus:—

EXAMPLE 3.—Multiply 42650 by 2300.

```
    42650
     2300
    -----
   15037
  10552
---------
122557000
```

Multiplication by 6 may be accomplished with the same result by its factors 2 and 3 in succession, and so with the factors of any other number.

Numbers which cannot be broken up into factors are called Prime Numbers,—3, 5, 7, 13, etc.

EXAMPLES.

	(1)	(2)	(3)	(4)
Multiply	3642	5340	4263	57312
By	231	132	246	6421
	3642	12700	32062	57312
	13346	20240	21314	136624
	7504	5340	10546	275450
	1107722	751300	1322022	434274
				465506552

DIVISION.

DIVISION is a short method of repeated subtraction; or, it is the method of finding how often one number, called the Divisor, is contained in another number, called the Dividend. The number which shows this is called the Quotient.

Thus the dividend 11 divided by the divisor 3 gives the quotient 3; and for this reason 3+3+3=11, and therefore if we subtract 3 from 11, and then a second 3 from the remainder 6, and then a third 3 from the remainder 3, nothing remains.

If, however, some number be left after the divisor has been taken as often as possible from the dividend, that number is called the Remainder; thus 10 divided by 3 gives a quotient 2, and a remainder 2; for, after subtract-

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ing 3 from 10 once, there is a remainder 5; after subtracting 3 a second time from the remainder 5, there is a remainder 2.

The sign $\div$, called By, or Divided by, placed between two numbers, signifies that the first figure is to be divided by the second.

Division is the opposite of multiplication. By the Multiplication Table, $3 \times 4 = 14$, and $14 \div 4 = 3$, or $14 \div 3 = 4$.

RULE FOR DIVISION WHEN THE DIVISOR IS A NUMBER NOT LARGER THAN SEVEN.

RULE. — Place the divisor and dividend thus: —

$$\text{divisor} \,)\, \underline{\text{dividend}}$$
$$\text{quotient}$$

Take off from the left hand of the dividend the least number of figures which make a number not less than the divisor. Find by the Multiplication Table how often the divisor is contained in this number; write the quotient under the units' figure of this number; if there is a remainder, affix to it, in the units' place, the next figure in the dividend, and proceed as before. Whenever there is no remainder, and the next figure does not contain the divisor, place a cipher in the next place to be filled in the quotient. If there be a remainder at the end of the operation, write it distinct from the quotient, *and write the divisor under it for a record.*

EXAMPLE 1.— Divide 1652 by 4.

By the Rule. $4)\underline{1652}$
$$352\tfrac{2}{4}$$

METHOD.—4 into 1 I cannot; therefore affix the next figure to the 1. Then 4 into 16 will go 3 times, leaving a remainder 2; write down the 3 under

the units' place of the number just dealt with, and to the remainder affix the next number in the dividend; then 4 into 25 will go 5 times, leaving a remainder 1. Write down five in the quotient; next affix the last number in the dividend to the remainder; then 4 into 12 will go twice, leaving 2 as a remainder. Write down the 2 under the dividend. Write the remainder and divisor separately.

EXAMPLES.

(1)	(2)	(3)	(4)
$245 \div 3$	$4321 \div 5$	$6732 \div 6$	$73215 \div 4$
$3\overline{)245}$	$5\overline{)4321}$	$6\overline{)6732}$	$4\overline{)73215}$
67	$703\frac{2}{5}$	1117	$16643\frac{1}{4}$

RULE FOR DIVISION WHEN THE NUMBER IS GREATER THAN EIGHT.

RULE. — Place the divisor and dividend thus: —

divisor) dividend (quotient

leaving space for the quotient to the right of the dividend. Take off from the left hand of the dividend a number not less than the divisor.

Find how many times the divisor is contained in this number. Place the number so found in the quotient; multiply the divisor by the quotient and bring down the product under the number taken off from the left of the dividend, and subtract.

On the right of the remainder bring down the next figure in the dividend. Find how many times the divisor is contained in this number. If it is less than the divisor, write a cipher in the quotient, and bring down another figure from the dividend and affix also to the remainder; repeat the process until all the figures in the dividend have been brought down.

EXAMPLE 1.—Divide 1432 by 31.

By the Rule.

$$\begin{array}{r} 31)1432(37\tfrac{23}{31} \\ 113\phantom{2(37\tfrac{23}{31}} \\ \hline 302\phantom{(37\tfrac{23}{31}} \\ 257\phantom{(37\tfrac{23}{31}} \\ \hline 23\phantom{(37\tfrac{23}{31}} \end{array}$$

METHOD.—143 is the least number taken from the left of the dividend into which 31 will go; for if 31 be × 3, the product will be 113, which is less than the number taken. Multiply the divisor by 3; place the product under the number from the dividend and subtract; the remainder is less than the divisor; ∴ place the multiplier as first figure in the quotient; bring down the next figure from the dividend and affix to the remainder. Find how many times the divisor is contained in the new dividend. Place the multiplier 7 as before in the quotient. Write down the product under the new dividend and subtract. There is a remainder 23. Write it in the quotient, and the divisor under it, for a record.

EXAMPLES.

(1)

14160÷21

$$\begin{array}{r} 21)14160(560 \\ 125 \\ \hline 146 \\ 146 \\ \hline \ldots 0 \end{array}$$

(2)

23651÷34

$$\begin{array}{r} 34)23651(552\tfrac{21}{34} \\ 214\phantom{51(552\tfrac{21}{34}} \\ \hline 225\phantom{1(552\tfrac{21}{34}} \\ 214\phantom{1(552\tfrac{21}{34}} \\ \hline 111\phantom{(552\tfrac{21}{34}} \\ 70\phantom{(552\tfrac{21}{34}} \\ \hline 21\phantom{(552\tfrac{21}{34}} \end{array}$$

(3)

$52765 \div 46$

$$\begin{array}{l} 46)52765(1103\tfrac{3}{46} \\ \underline{46} \\ 47 \\ \underline{46} \\ 165 \\ \underline{162} \\ 3 \end{array}$$

(4)

$32651 \div 123$

$$\begin{array}{l} 123)32651(245\tfrac{52}{123} \\ \underline{246} \\ 605 \\ \underline{514} \\ 711 \\ \underline{637} \\ 52 \end{array}$$

SECTION 3.

DEFINITIONS.

A Unit is a predeterminate quantity of weight, length, capacity, or value.

For purposes of calculation, the Unit occupies a position immediately to the left of a point.

Superior Denominations are all those units which appear to the left of the point.

Inferior Denominations are all those units which appear to the right of the point.

THE POINT.

A Point is a sign so placed as to distinguish and to separate Superior Units from Inferior Units for the purpose of determining their value. Thus 1.1 means one unit plus one eighth of one unit.

THE FRACTION.

A Fraction is an undetermined portion of a Unit remaining after units have been subdivided to the lowest convenient denomination.

Example. — Three men wish to apportion 4 dollars among themselves as nearly equal as possible. The calculation is carried out by the rule for division; thus, —

$$\begin{array}{r} \$\ \ ¢ \\ 3\overline{)4.00} \\ \hline 1.25\frac{1}{3} \end{array}$$

Since 400 is not a multiple of 3, and the nearest number which is a multiple of 3 is 377, there must be a remainder of 1 cent, which must be dealt with in a manner to be subsequently shown.

REDUCTION.

Reduction is carried out by the use of the point, termed Inspection; thus 400 cents=$4.00.

tons lbs. oz.	q. pt. in.
32126 oz.=3.212.6	265 inches Liquid=2.6.5
	yd. ft. in.
	265 " Linear=2.6.5

Thus—

E. $ c	
17464.5.32	American currency.
miles yd. ft. in.	
174.645.3.2	Linear measure.
yd. ft. in.	
1746.45.32	Square measure.
yd. ft. in.	
17.464.532	Cubic measure.
tank pk. q. p. i.	
17.464.5.3.2	Capacity liquid measure.
ton lb. oz. dr. sc. gr.	
1.746.4.5.3.2	Weights.
days hr. m. s.	
17.46.45.32	Time.
Revolution ° ′ ″	
17.46.45.32	Circle.

Table of Weights.

10 Grains	make	1 Scruple.
10 Scruples	"	1 Drachm.
10 Drachms	"	1 Ounce.
10 Ounces	"	1 Pound.
1000 Pounds	"	1 Ton.

Note.—The pound avoirdupois is the unit of weight.

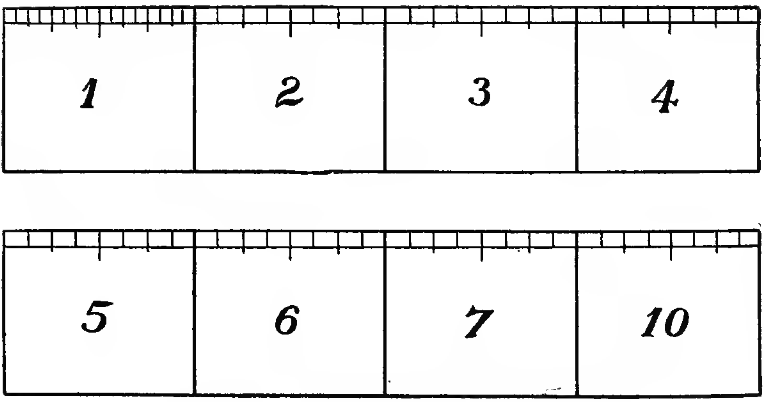
Table of Measures of Capacity.

10 Minims	make	1 Cubic Inch.
10 Cubic Inches	"	1 Pint.
10 Pints	"	1 Quart.
10 Quarts	"	1 Peck.
1000 Pecks	"	1 Tank.

Table of Linear Measure.

10 Inches	make	1 Foot.
10 Feet	"	1 Yard.
1000 Yards	"	1 Mile.

Note.—The present unit, the Inch, is to be retained.



ONE-FOOT RULE (SHOWN IN TWO SECTIONS).

Currency.

10 Mills	make	1 Cent.
10 Cents	"	1 Bit.
10 Bits	"	1 Dollar.
10 Dollars	"	1 Eagle.

Note. — The present Dollar is the unit to be retained.

Convenient American Coins.

Gold.	Eagle=Eight (10) dollars	To be minted.
Silver.	Dollar=Etred (100) cents	Present dollar.
"	Half-dollar=Four-et (40) cents	" 50 cents.
"	Quarter-dollar=Two-et (20) cents	" 25 cents.
"	Eighth-dollar=Eight (10) cents .	To be minted.
Nickel.	Four (4) cents.	"
Copper.	Two (2) cents.	"
"	One (1) cent	"

Value of Gold Decimal Coin in Octimal Figures.

Five	($5) pieces=	Five (5) dollars.
Ten	($10) pieces=	Et-two (12) dollars.
Twenty	($20) pieces=	Two-et four (24) dollars.

Measures of Time.

100 Seconds	make	1 Minute.
100 Minutes	"	1 Hour.
100 Hours	"	1 Day.

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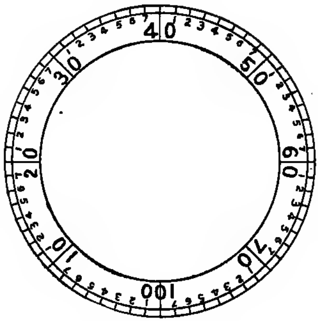
SUBDIVISIONS OF CIRCLE.

100″ (Seconds)	make	1′ (Minute).
100′ (Minutes)	"	1° (Degree).
100° (Degrees)	"	1^R (Circle).



COMPOUND NUMBERS.

COMPOUND NUMBERS are those numbers which contain both Superior and Inferior units.

COMPOUND ADDITION.

RULE.—Place the numbers under each other, units under units, eights under eights, etc., so that the points shall be directly under each other. Add as in whole numbers, and place a point in the result immediately under the points in the numbers above.

EXAMPLE. — Add together 431.64, 56.200, .0321.

By the Rule.

$$\begin{array}{r} 431.64 \\ 56.200 \\ .0321 \\ \hline 510.0721 \end{array}$$

COMPOUND SUBTRACTION.

RULE. — Place the less number under the greater, units under units, eights under eights, etc., so that the points shall be directly under each other. If there be fewer figures in the upper number to the right of the point than there are figures in the lower number to the right of the point, add ciphers to the upper number until the numbers of figures are equal. Then subtract as in whole number, placing a point immediately under the point above.

EXAMPLE. — Subtract 31.2652 from 53.21.

By the Rule.

$$\begin{array}{r} 53.2100 \\ 31.2652 \\ \hline 21.7226 \end{array}$$

COMPOUND MULTIPLICATION.

RULE. — Multiply the numbers together as if they were whole numbers, and point off in the product as many places as there are in both the multiplicand and multiplier. If there are not sufficient figures, prefix ciphers.

EXAMPLE. — Find the product of .2×.3; also, 1.4×2.3.

By the Rule.

$$\begin{array}{r} .2 \\ .3 \\ \hline .06 \end{array} \qquad\qquad \begin{array}{r} 1.4 \\ 2.3 \\ \hline 44 \\ 30 \\ \hline 3.44 \end{array}$$

COMPOUND DIVISION.

When the number of inferior places in the dividend exceeds the number of inferior places in the divisor.

Rule. — Divide as in whole numbers, and point off in the quotient a number of inferior places equal to the excess of the number of inferior places in the dividend over the number of inferior places in the divisor; if there are not figures sufficient, prefix ciphers, as in Multiplication.

Example 1. — Divide 23.006 by 2.35. .0023006 by 2.35.

```
By the Rule.  2.35)23.006(7.6      2.35).0023006(.00076
                   21 13                  2113
                   -----                  ----
                    1 656                  1656
                    1 656                  1656
                    -----                  ----
                                           ....
```

When the number of inferior places in the dividend is less than the number of inferior places in the divisor.

Rule. — Affix oiphers to the dividend until the number of inferior places in the dividend equals the number of inferior places in the divisor; the quotient up to this point of the division will be a whole number. If there be a remainder, and the division be carried on further, the figures after this point in the quotient will be inferior units.

Example 2. — Divide 2300.6 by .235.

```
By the Rule.  .235)2300.600(7600.
                   2113
                   ----
                    1656
                    1656
                    ----
                    ....00
```

RULE. — Before dividing, affix ciphers to the dividend to make the number of inferior places in the dividend exceed the number of inferior places in the divisor by three. If we divide up to this point, the quotient will contain three inferior places.

EXAMPLE 3. — Divide 2301.2 by .235 to 3 inferior places.

```
.235)2301.200000(7601.502
      2113
      ----
       1662
       1656
       ----
       ···400
          235
          ---
          1430
          1421
          ----
           ··700
              472
              ---
              206
```

REPETENDS.

The sign ˙ applied to a number means that the number is recurring, or a Repeater; thus, $.\dot{2}$. Repetends are those numbers which periodically recur in Compound Division and may be extracted to the lowest required denomination by merely affixing them to the quotient; thus, $4 \div 3$ gives the quotient $1.\dot{2}\dot{5}$; and the quotient may be extended thus, $1.\dot{2}\dot{5}\dot{2}\dot{5}\dot{2}\dot{5}$.

SECTION 4.

RATIO AND PROPORTION.

The relation of one number to another in respect of magnitude is called Ratio.

The Ratio of one number to another is denoted by placing a colon between them. Thus the ratios of 3 to 14 and 14 to 3 are denoted by 3:14 and 14:3.

When two Ratios are equal, they are said to form a Proportion, and the four numbers are called Proportionals. Thus the ratio of 3 to 14 and 6 to 30 are equal. The Ratios being equal, Proportionals exist among the numbers 3, 6, 14, 30.

The existence of Proportion between the numbers 3, 6, 14, 30, is denoted thus: 3:6 :: 14:30.

If four numbers be proportionals when taken in a certain order, they will also be proportionals when taken in a contrary order; for instance, 3, 6, 14, 30, are proportionals, as already shown; and contrariwise, 30:14 :: 6:3.

The two numbers which form a Ratio are called its terms; the former term is called the Antecedent; the latter, the Consequent.

Because $3\times2=6$, and $14\times2=30$, ∴ are those numbers proportional. Again: because the Antecedent of each when multiplied by the Consequent of the other produces a like number, ∴ are they proportional.

If any three terms of a proportion be given, the remaining term may always be found. For since in any Proportion,

$$\text{1st term} \times \text{4th term} = \text{2d term} \times \text{3d term},$$
$$\therefore \text{1st term} = \text{2d} \times \text{3d} \div \text{4th};$$
$$\text{2d term} = \text{1st} \times \text{4th} \div \text{3d};$$
$$\text{3d term} = \text{1st} \times \text{4th} \div \text{2d};$$
$$\text{4th term} = \text{2d} \times \text{3d} \div \text{1st}.$$

EXAMPLE 1.—Find the 4th term in the proportion 3, 6, 14.

$3:6::14:\text{4th term}$; $\therefore$ 4th term $=6\times14\div3=30$, Ans.

RULE OF THREE.

THE RULE OF THREE is a method by which we are enabled, from three numbers which are given, to find a fourth, which shall bear the same ratio to the third as the second to the first; in other words, it is a rule by which, when the three terms of a proportion are given, we can determine the fourth.

RULE.—Find out, of the three quantities which are given, that which is of the same kind as the fourth or required quantity, or that which is distinguished from the other terms by the nature of the question. Place this quantity as the third term of the proportion.

Now consider whether, from the nature of the question, the fourth term will be greater or less than the third; if greater, then put the larger of the other two quantities in the second term and the smaller in the first term; but if less, put the larger in the first term and the smaller in the second term.

Multiply the second and third terms together, and divide by the first. The quotient will be the answer to the question.

EXAMPLE 1.—If oats are worth \$24 per ton, how much can be bought for \$50? Since $\overset{\text{ton lb.}}{1.000}$ is of the same kind as the required term,—viz., tons,—we make $\overset{\text{ton lb.}}{1.000}$ the third term. Since \$50 will buy more tons than \$24, we make \$50 the second term and \$24 the first term:

$$\$24:\$50::\overset{\text{ton lb.}}{1.000}:\text{4th term.}$$

$$24\div50\times1.000=\text{4th term}=\overset{\text{tons lb.}}{2.000}.$$

EXAMPLE 2.—A has a contract to dig a ditch in 2 days. He has 4 men employed, but at present progress it will take 4 days to complete the work. How many men should he have employed to finish the contract on time?

$$\overset{\text{days}}{2}:\overset{\text{days}}{4}::\overset{\text{men}}{4}:\text{4th term.}$$

$2\div4\times4=10$, the number of men required.

PRACTICE.

SIMPLE PRACTICE.

In this case the given number is expressed in the same denomination as the unit whose value is given; as, for instance, 21 lb. @ \$1.20 per lb.

RULE.—Multiply the numbers together without reference to their names, and point off the terms of the several denominations in the product.

EXAMPLE.—43 lb. of sugar @ 7¢ per lb. The product will be cents; ∴ $43\times7=365$ cents, =\$3.65¢.

Compound Practice.

In this case the given number is not wholly expressed in the same denomination as the unit whose value is given; as, for instance, 1 ton 235 lb. @ \$63 per ton.

Rule.—State the question thus: If one ton cost \$63.00, what will 1,235 lb. cost?

Multiply the second and third terms together without reference to their names; thus:—

$$1235 \times 6300 = 10250\cancel{7}\cancel{0}\cancel{0}.$$

From the right of the product cut off a number of figures which is equal to the number of inferior units in the third term. The remaining figures will be the value required, of the same name as the second term. Thus, \$102.50¢, Ans.

INTEREST.

Interest (Int.) is the sum of money paid for the loan or the use of some other sum of money, lent for a certain time at a fixed rate; generally at so much for each \$100 for one year.

The money lent is called the Principal.

The interest on \$100 for a year is called The Rate per Cent.

The principal + the interest is called the Amount.

Interest is divided into Simple and Compound. When interest is reckoned only on the principal or sum lent, it is Simple Interest.

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When the interest at the end of the first period, instead of being paid by the borrower, is retained by him and added as principal to the former principal, interest being calculated on the new principal for the next period, and this interest, again, instead of being paid, is retained and added on to the last principal for a new principal, and so on, it is Compound Interest.

Simple Interest.

To find the interest on a given sum of money at a given rate per cent for a year.

Rule. — Multiply the principal by the rate per cent and divide the product by 100.

Example. — Find the simple interest of \$240 for one year, at 7 per cent per annum.

By the Rule. $\$240.00 \times 7 \div 100 = \$21.40$.

Compound Interest.

To find the Compound Interest of a given sum of money at a given rate per cent for any number of years.

Rule. — At the end of each year, add the interest for that year to the principal at the beginning of it; this will be the principal for the next year; proceed in the same way as far as may be required by the question. Add together the interests so arising in the several years, and the result will be the compound interest for the given period.

Example. — Find the Compound Interest and Amount of \$600 for 3 years, at 6 per cent per annum.

$600 \times 6 \div 100 = \$44$ Int. 1st yr.

$600 + 44 = 644 \times 6 \div 100 = \$47.30$ Int. 2d yr.

$644 + 47.30 = 713.30 \times 6 \div 100 = \$53.0420$ Int. 2d yr.

$713.30 + 53.0420 =$ Ans. \$766.34.20.

$\therefore$ Compound interest $= \$53.0420 + \$47.30 + \$44 = \$166.3420$.

Amount, $\$600 + \$166.3420 = \$766.342$.

SECTION 5.

SQUARE ROOT.

The Square of a given number is the product of that number multiplied by itself. Thus 6×6, or 44, is the square of 6, or $44=6^2$.

The Square Root of a given number is a number which, when multiplied by itself, will produce the given number.

The Square Root of a number is denoted by placing the sign $\sqrt{}$ before the number. Thus $\sqrt{44}$ denotes the square root of 44, so that $\sqrt{44}=6$. The sign ^{2} placed above the number a little to the right denotes that the number is to be squared; thus, $6^2=44$.

TABLE 1.

1=	$\sqrt{1}$
2=	4
3=	11
4=	20
5=	31
6=	44
7=	61
10=	100

TABLE 2.

1=	$\sqrt{1}$
$\dot{2}$=	$\dot{4}$
$\dot{4}$=	$2\dot{0}$
$\dot{1}\dot{0}$=	$\dot{1}0\dot{0}$
$\dot{2}\dot{0}$=	$\dot{4}0\dot{0}$
$\dot{4}\dot{0}$=	$2\dot{0}0\dot{0}$
$\dot{1}\dot{0}\dot{0}$=	$\dot{1}0\dot{0}0\dot{0}$
$\dot{2}\dot{0}\dot{0}$=	$\dot{4}0\dot{0}0\dot{0}$

NOTE.—The squares of all those numbers which are in geometrical progression appear with the order of their indices reversed, and their roots may be extracted by mere inspection.

A square figure will always contain the square of a number of square units.

The square root of a number is one side of a square with area equal to the number.

∴ A square containing 44 square units has a side containing 6 linear units of the same value as one side of each of the 44 square units contained in the given square.

RULE TO EXTRACT THE SQUARE ROOT OF A GIVEN NUMBER.

RULE.—Place a dot over the units' place of the given number; and thence over every second figure to the left of that place; and thence over every second figure to the right, when the number contains inferior units, annexing a cipher when the number of inferior places is odd; thus dividing the number into periods. The number of dots over the superior and inferior units respectively will show the superior and inferior places in the root.

From Table 1 take the index which is nearest (but less) to the first period, and write its root in the first place in the root. Now consider whether the first period is nearer the index already taken, or nearer the index which comes next in the table. If nearer the index taken, try a number less than four for the second place in the root; now square the number for trial. If the product is greater than the given number, try a number which is less; repeat the operation until all the places in the root are filled.

EXAMPLE 1.—Find the Square Root of 1261.04.

$$\dot{3}\dot{2}.\dot{2}=\sqrt{1\dot{2}6\dot{1}.0\dot{4}}$$

$3\sqrt{11}$	$4\sqrt{20}$
32	33
32	33
64	121
116	121
1244	1331

METHOD.—From Table 1 take the Index 11; the next one, 20; apply the root of the 11 to first place; first period is near 11; ∴ try 32^2, also 33^2; the product 1244 is nearest the given number; ∴ write 2 in the root; 32^2 is much nearer the given number than 33^2; ∴ try 322.

```
   322
   322
   ---
   644
  644
 1166
------
126104
```

$322 \times 322 = 126104.$

Square root of $1261.04 = 32.2$.

A RECTANGLE.

A mixed number of square units will always be contained in a Parallelogram, one of whose dimensions shall be a root number, as in Table 2.

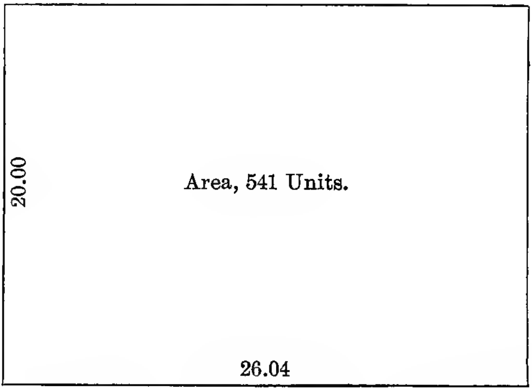
Rule.—To conform the given number to a parallelogram, divide the given number by the selected dimension. The quotient will be a perpendicular. The divisor and the quotient will be the required dimensions.

Example 1.—Given one side and the area of a Parallelogram, to find the other side, one side to be 20, the area to be 541.

By the Rule. $541 \div 20 = 26.04$;

∴ one dimension $= 20$;

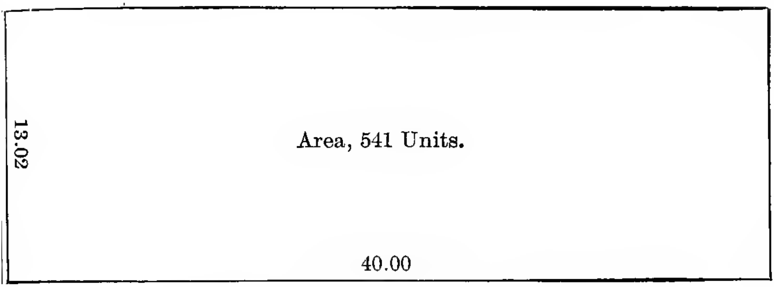
" " $= 26.04$.



THIS DIAGRAM IS ONE EIGHTH ACTUAL DIMENSIONS.

Example 2.—Conform 541 to the required Parallelogram, one side to be 40.

By the Rule. $541 \div 40 = 13.02$;
$\therefore$ one dimension $= 40$;
" " $= 13.02$.



THIS DIAGRAM IS ONE EIGHTH ACTUAL DIMENSIONS.

CUBE ROOT.

The Cube of a given number is the product which arises from multiplying that number by itself, and then multiplying the result again by the same number. Thus $6 \times 6 \times 6$, or 330, is the cube of 6; or $330 = 6^3$.

The Cube Root of a given number is a number which, when multiplied into itself, and the result is again multiplied by it, will produce the given number. Thus 6 is the Cube Root of 330; for $6 \times 6 = 44$, and $44 \times 6 = 330$.

The Cube Root of a number is denoted by placing the sign $\sqrt[3]{\ }$ before the number. Thus $\sqrt[3]{330}$ denotes the Cube Root of 330; so that $\sqrt[3]{330} = 6$.

TABLE 1.		TABLE 2.	
1=	$\sqrt[3]{1}$	1=	$\sqrt[3]{1}$
2=	10	2=	$1\dot{0}$
3=	33	4=	$10\dot{0}$
4=	100	10=	$\dot{1}000$
5=	175	20=	$1\dot{0}000$
6=	330	40=	$10\dot{0}00\dot{0}$
7=	527	100=	$\dot{1}00\dot{0}00\dot{0}$
10=	1000	200=	$1\dot{0}00\dot{0}00\dot{0}$

NOTE.—The Cubes of all those numbers which are in geometrical progression occur in ciphers with the index 1. Their roots may be extracted by mere inspection.

All the indices will reappear when their roots are bisected continuously.

Cube Root is the number of units of measure contained in a line which is one edge of a Cube.

The Cube Root of a number can only be found when the given number is the cube of the number of units in the base.

RULE FOR EXTRACTING THE CUBE ROOT OF A GIVEN NUMBER.

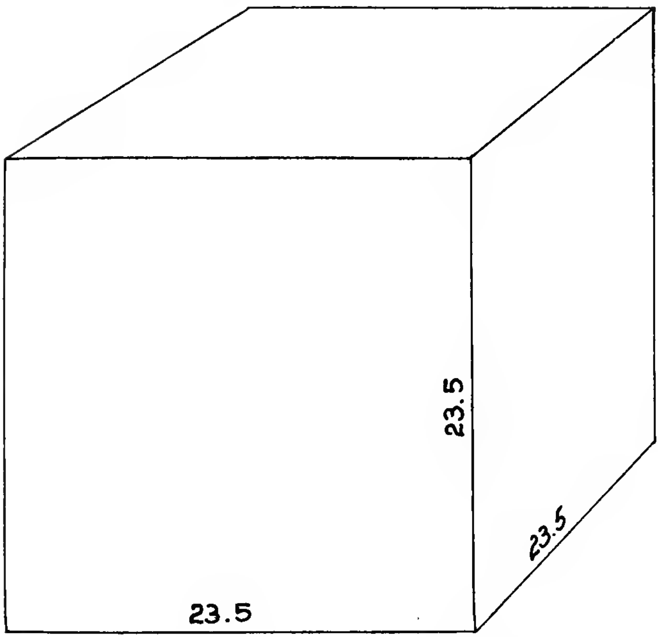
RULE.—Place a dot over the units' place of the given number, and thence over every third figure to the left, and thence over every third figure to the right, when the number contains inferior units, affixing one or two ciphers when necessary to make the number of inferior places a multiple of three, thus dividing the given numbers into periods. The number of dots over the whole number and inferior units, respectively, will show the whole number and inferior places in the root.

Place the sign $\sqrt[3]{}$ over the given number. Point over the same number of places to the left of the sign the same number of dots that there are periods in the given number, in the form of a divisor.

Find among the indices the number which is nearest (but below) the number in the first period, and place the corresponding root number in the first place in the divisor.

Now consider whether the index just applied is nearer the given number than is the index which is nearest above or greater than the given number.

If the index applied is the nearest, add a number which is less than four, now cube the new number, and if the product be less than the given number, write down the figure in the second place in the root, but if it be greater, try a lower number. This operation is to be repeated until all the places in the root are filled.



THIS CUT SHOWS ONE EIGHTH OF THE ACTUAL DIMENSIONS.

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EXAMPLE.—Extract the Cube Root of 16606.305.

$$23.5_{1}\ \overline{16606.305}$$

By the Rule. $2_{1}\ \overline{10}$ $\qquad$ $3_{1}\ \overline{33}$

23	24	235
23	24	235
71	120	1421
46	50	727
551	620	472
23	24	60111
2073	3100	235
1322	1440	360555
15313	17500	220333
		140222
		16606305

METHOD.—From Table 1 take the index 10; apply its root to the first place; the index applied is nearer the first period than 33; ∴ try 23^3, also 24^3, then write 3 in the second place. The given number is nearer 24^3; ∴ try 235^3.

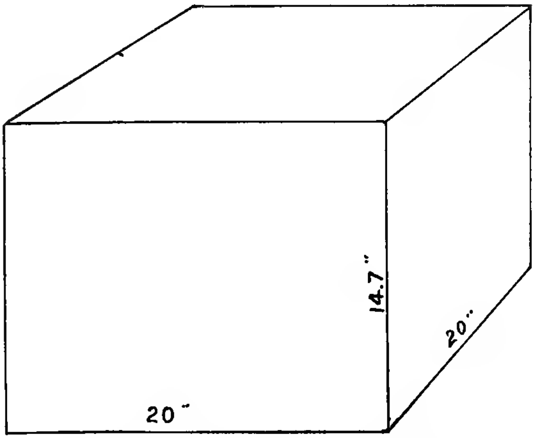
RECTANGLE PARALLELOPIPED.

A mixed number of cubical units will always be contained in a Parallelopiped, two of whose dimensions shall be a root number in Table 2.

RULE.—To conform a given number to a Parallelopiped, multiply the selected dimensions together and divide the given number by the product. The quotient will be the third dimension of a Parallelopiped.

EXAMPLE 1.—Conform 6340 to a Parallelopiped.

By the Rule. Selected dimensions=20 each;
∴ 20×20=400=area of one side;
6340÷400=third dimension=14.7;
∴ one dimension=20;
Second dimension=20;
Third dimension=14.7.



THIS CUT SHOWS ONE EIGHTH OF THE ACTUAL DIMENSIONS.

EXAMPLE 2.—Selected dimensions are: one dimension, 10; one dimension, 40;
∴ 10×40=400=area of one side;
6340÷400=third dimension=14.7;
∴ one dimension=10;
Second dimension=40;
Third dimension=14.7.

DERIVATION OF RADIX.

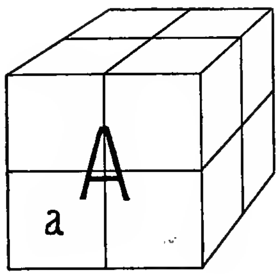
The Radix is suggested by the number of parts into which the Cube may be subdivided, when all its dimensions are bisected. Hence the number of symbols in the Octimal Notation permit the same mechanical treatment as does the Cube or any other regular solid; ∴ since the third mechanical subdivision of the Cube results in 10 Cubes, each a counterpart of the original in the ratio of 10 to 1, therefore will the third bisection of the radix result in symbols the counterpart of the original in the ratio of 10 to 1.

ELEVATION OF DENOMINATIONS.

Whole numbers are raised to the next higher denomination by affixing a cipher; thus, $35 \times 10 = 350$.

Whole numbers are reduced to the next lower denomination by introducing a point; thus, $35 \div 10 = 3.5$.

∴ let A represent a cube, and a, a cube, then if the weight of A=35 lb., a=3.5 lb.



ELEVATION OF CUBES AND THEIR ROOTS.

For each time a Root is multiplied by two, raise cube number one denomination by affixing a cipher.

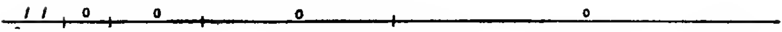
For each time a Root is bisected, reduce its cube number one denomination by pointing off one figure from the right of the whole number.

CALCULATION BY CONSTRUCTION.

A Cube, Sphere, or Parallelopiped may be reduced to convenient limits for calculation by simply bisecting its diameter to the lowest convenient denomination.

Rule.—For each time the three dimensions are bisected, write down one cipher, and for the last place apply the index 1, or x. When the value of the index is found, replace the index by that value.

Example.—A cube of bricks is 140.0 feet long at base. How many bricks does it contain? and what is its value at \$7.00 per etand bricks?



By the Rule.

2)140.0
 2)60.0
 2)30.0
 2)14.0
  6.0=6.0 ft.=Index.

Ans. { Number, 12,530,000.
Value, 12,530,000×\$7.00=\$112,550.00.

The Index is found to contain 1253 bricks; therefore to the sum so found affix four ciphers.

SECTION 6.

LONGITUDE AND TIME.

The Earth's Circle is subdivided, commencing opposite the meridian of Greenwich.

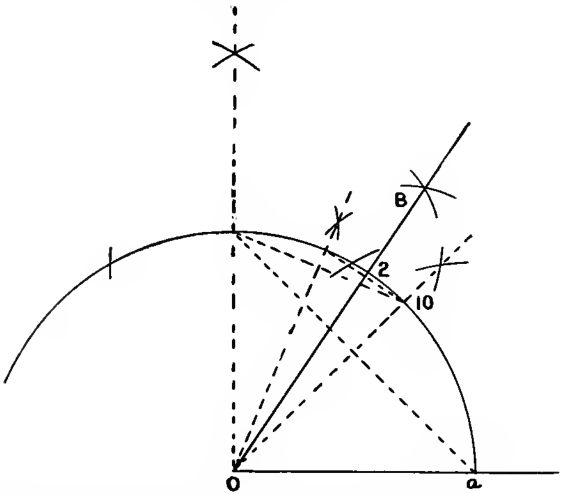
The Dial is subdivided to correspond from midnight.

Longitude is therefore always west.

Difference in time gives Longitude.

EXAMPLE.—Meridian of Departure=0. The sun passes overhead 40 hr. 21′ later. What is your Longitude? Ans. 40° (degrees) 21′ (minutes).

TO DETERMINE AN ANGLE.



EXAMPLE.—Required the Angle A O B.
Ans. Angle=12 degrees.

RULE. — At point of intersection raise a perpendicular. Bisect the right angle. The value of the new angle = 10. Should the angle exceed 10, bisect again and continue to the ratio of 1 to 10, repeating the operation if necessary.

When the Angle is less than 10, the operation is similar.

AREA OF CIRCLE IN TERMS OF SQUARE UNITS.

The diameter of a circle is to its circumference as 1 is to 3.1. — Incorrect.

The square of the diameter of a circle is to its area as 1 is to $.6\dot{2}$.

∴ Multiply the square of the diameter by $.6\dot{2}2\dot{2}$.

EXAMPLE. — Required the area of a circle whose diamter is 16.2 inches.

By the Rule. $16.2^2 = 313.04 \times .6222 = 237.421710$ inches.

SECTION 7.

MISCELLANEOUS.

1 Meter $=47.2753412$ Inches, nearly.

$\therefore$ A Centimeter is to an Inch as $1:2.43$; $\therefore$ Inches $\times 2.43=$ number of Centimeters.

An Inch is to a Centimeter as $1:.311$; $\therefore$ Cm. $\times .311=$ number of Inches.

NOTE.—This comparison is sufficiently accurate for all practical purposes in measurements up to one foot.

EXAMPLE.—Required the value of 6.2 Inches in Centimeters.

By the Rule. $\overset{\text{in.}}{6.2}\times 2.43=17.723$ Centimeters.

TRANSFORMATION OF NUMBERS.

Decimal numbers may be transformed into Octimal numbers.

RULE.—Divide by eight continuously; the remainders will be the Octimal figures.

EXAMPLE.—1631 Decimal $=3137$ Octimal.

By the Rule.

```
8)1631
  8)203(7
    8)25(3
       3(1
```

NOTE.—The operation is carried on in the decimal system.

Octimal numbers may be transformed into Decimal numbers.

RULE.—Divide by et-two continuously; the remainders will be the Decimal numbers.

EXAMPLE.—15356 Octimal=6894 Decimal.

By the Rule. 12)15356
12)1261(4
12)104(11=9
6)10=8

DECIMAL FRACTIONS.—TO TRANSFORM INTO INFERIOR UNITS.

RULE.—Multiply by eight continuously. The figures to the left of the point in each product will be the Octimal Inferior Units required.

EXAMPLE.—.125=.1.

By the Rule. .125
8
1.000

NOTE.—When all the figures in the product are below the point, supply a cipher in the units' place.

EXAMPLE.—.0625=.04.

By the Rule. .0625
8
0.5000
8
4.0000

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DECIMAL NUMBERS AND OCTIMAL INFERIOR UNITS.

ADDITION.

RULE.—Add the octimal inferior units together, carrying eights. Carry the superior units, and add to the decimal numbers above the point. Continue in the decimal system.

	Dec. Oct.		Dec. Vulgar Fractions.	
EXAMPLE.—	29.3	inches	$=29+\frac{3}{8}$ inches.	
	29.6	"	$=29+\frac{3}{4}$ "	
	29.23	"	$=29+\frac{1}{4}$ "	$+\frac{3}{64}$.
	29.76	"	$=29+\frac{7}{8}$ "	$+\frac{6}{64}$.
	118.31		$118+\frac{3}{8}$ inches	$+\frac{1}{64}$.

MULTIPLICATION.

Dec. Oct.	Dec. Vul. Frac.
$118.31\times 4=$	$118+\frac{3}{8}+\frac{1}{64}\times 4$
4	4
473.44	$473+\frac{1}{2}+\frac{4}{64}$

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